Abstract—This letter presents a method of measuring the permittivity of a liquid contained inside a waveguide, having a four-boundary arrangement. Previous methods employ an expression for the total transmission through a four boundary cell. We present a new method in which the permittivity is obtained by de-embedding the S-parameters of the sample from the S-parameters measured for the whole assembly. The advantage of this method is that it can also be used when the permittivity of the material housing the liquid is not accurately known.

Index Terms—Complex permittivity measurement, liquids, waveguides.

I. INTRODUCTION

The use of transmission and reflection, measured in a transmission line filled with a sample under test (SUT) was first employed by Nicholson–Ross [1] then Weir [2] in time frequency domain, respectively. The problems associated with their solution for complex permittivity is well documented in the literature [3], [4]. Baker–Jarvis [5] has shown that iterative methods of solutions eliminate these problems.

The expressions for the total transmission and reflection through a series of dielectrics inside a waveguide as shown in Fig. 1 are given in (1) and (2), respectively, with \( Z_0 \), \( \gamma_0 \), \( Z_s \), and \( \gamma_s \) as the wave impedance and propagation constant of an air filled and sample filled waveguide, respectively, [1], [2], [6]–[13] assuming only the dominant TE\(_{20}\) is present. The multislab assembly as in Fig. 2 is used due to the need to house the liquid with a solid on either side

\[
S_{21} = \frac{T_S (1 - T_S^2)}{1 - T_S^2 T_S^2} = \frac{4 \gamma_S \gamma_0}{(\gamma_S + \gamma_0)^2 e^{2\gamma_s l_S} - (\gamma_S - \gamma_0)^2 e^{-2\gamma_s l_S}} \tag{1}
\]

\[
S_{11} = \frac{T_S (1 - T_S^2)}{1 - T_S^2 T_S^2} = \frac{(\gamma_S^2 - \gamma_0^2) e^{2\gamma_s l_S} - (\gamma_S^2 - \gamma_0^2) e^{-2\gamma_s l_S}}{(\gamma_S + \gamma_0)^2 e^{2\gamma_s l_S} - (\gamma_S - \gamma_0)^2 e^{-2\gamma_s l_S}} \tag{2}
\]

and then (3) and (4), shown at the bottom of the next page, where \( \Gamma_1 = (Z_P - Z_0)/(Z_P + Z_0) \), \( \Gamma_2 = (Z_s - Z_P)/(Z_s + Z_P) \), \( T_1 = e^{-\gamma_P l_P} \), and \( T_2 = e^{-\gamma_s l_S} \)

In the existing literature, (1) has been used [10]–[12] i.e., completely ignoring the properties of the material used for the liquid housing. However, according to [8], the effect of the housing is only negligible if its thickness is half wavelength at the centre frequency of the measurement. Olivera [13] and Bois et al. [14] do not make this assumption but make use of (3) assuming \( \varepsilon_p^* \) is accurately known.

II. THEORY

According to [15] the ABCD matrix \( [T_F] \) for the Perspex filled length of waveguide as shown in Fig. 3 can be written as a function of the propagation constant. It is also well known that the ABCD matrix of the entire Perspex-SUT-Perspex assembly \( [T_F] \) can also be related to the S-parameters measured for the whole assembly. The total ABCD matrix is \( [T_F] = [T_F][T_S][T_F] \) hence from which: \( [T_S] = [T_F][T_F]^{-1} \). From the ABCD matrix of the SUT \( ([T_S]) \) the S matrix of the sample only can then be obtained directly as

\[
S_{11S} = \frac{A_S + B_S Z_0^{-1} - C_S Z_0 - D_S}{A_S + B_S Z_0^{-1} + C_S Z_0 + D_S} \tag{5a}
\]

\[
S_{21S} = \frac{2}{\frac{2(A_S D_S - B_S C_S)}{A_S + B_S Z_0^{-1} + C_S Z_0 + D_S}} \tag{5b}
\]

\[
S_{12S} = \frac{-A_S + B_S Z_0^{-1} - C_S Z_0 + D_S}{A_S + B_S Z_0^{-1} + C_S Z_0 + D_S} \tag{5c}
\]

\[
S_{22S} = \frac{-A_S + B_S Z_0^{-1} - C_S Z_0 + D_S}{A_S + B_S Z_0^{-1} + C_S Z_0 + D_S} \tag{5d}
\]
We therefore obtain the S-matrix of the liquid only so that it resembles Fig. 1 and hence (1) and (2) for a dielectric material completely filling a waveguide is valid and can be used as an option assuming \( \varepsilon_p^* \) is known accurately. If \( \varepsilon_p^* \) is not accurately known it can be left as an additional unknown. The underlying assumption in both cases is that the length of the solid dielectric on either side which may not necessarily be of equal lengths as well as the liquid SUT (assumed homogenous) must be known in mm to an accuracy of one decimal place.

III. SAMPLE HOLDER DESIGN

The sample holder assembly is as shown in Fig. 3 and consists of two sections. The top is a short length of Perspex filled length of waveguide inside a flange of the same thickness.

The bottom section consists of a length of waveguide with identical flange on each end. Only one end however is filled with Perspex leaving room to place the liquid sample through the open end. The actual length holding the SUT is made very small in order to eliminate measurement uncertainties which result in measurement made if the \( S_{21} \) is less than \(-40 \) db [9], [14] and also to avoid half wavelength effects. The first section is then placed atop the first forming a Perspex-SUT-Perspex assembly. Physical discontinuity due to misalignment and air gaps are minimized using specially machined pins to fit the screw hole of the flange.

The S parameters of the entire assembly are then measured using an Agilent 8510 Network analyzer.

IV. COMPUTATION OF PERMITTIVITY

The permittivity of the SUT is obtained by solving non linear (6a)–(6d) iteratively

\[
\begin{align*}
\text{(6a)} & \quad f(x_1) = \text{Re}(\tau_1 - S_{21S}) \\
\text{(6b)} & \quad f(x_2) = \text{Im}(\tau_1 - S_{21S}) \\
\text{(6c)} & \quad f(x_3) = \text{Re}(\tau_2 - S_{21T}) \\
\text{(6d)} & \quad f(x_4) = \text{Im}(\tau_2 - S_{21T})
\end{align*}
\]

Only (6a) and (6b) are sufficient if \( \varepsilon_p^* \) is accurately known, otherwise, two additional unknowns can be solved for by including (6c) and (6d) where \( \tau_1 \) and \( \tau_2 \) are the equations in (1) and (3), and \( S_{21S} \) and \( S_{21T} \) are from (5b) and the measured network analyzer values, respectively. The roots \( x_1 \rightarrow x_4 \) of the equations solved iteratively represent the unknowns.

The iterations continue until convergence which is reasonably quick. Initial estimates can be obtained using Debye’s approximations at the starting frequency, with subsequent iterations using the values determined at the previous frequency. Initial estimates for \( \varepsilon_p^* \), \( \varepsilon_r^* \), and \( \varepsilon_d^* \) used are 50, 20, 2, and 0 in that order. A Matlab program was written and it employs one of the software’s in-built functions: fsolve. This was used to obtain permittivity of both the sample and the Perspex material for 201 frequency points.

V. MEASUREMENT AND RESULTS

The Calibration used for the measurements of the entire unit was the full two port calibration with reference plane set at the end on the Perspex holder on either side of the assembly as shown in Fig. 2.

Tap water and 0.17 Molar saline solutions were measured at room temperature using an X-band waveguide. The results for both liquids obtained using (6a) and (6b) only is presented in Fig. 4. An average value of \( \varepsilon_p^* = 2.55 - 0.025j \) measured independently for the X-band frequency range is used. As expected the real part of the permittivity of the saline solution is less than that measured for pure water, while the imaginary part is greater.

The real and imaginary part of the complex permittivity of tap water obtained using the simultaneous method [solving

\[
\begin{align*}
S_{21} &= \frac{(1 - \Gamma_1^2 - \Gamma_2^2 + \Gamma_1\Gamma_2)(T_1^2T_2)}{1 + 2\Gamma_1\Gamma_2 T_1^2 - \Gamma_2^2 T_2^2 - 2\Gamma_1\Gamma_2 T_1 T_2^2 + \Gamma_1^2 T_2^2 T_1 - \Gamma_1^2 T_1^2 T_2} \\
S_{11} &= \frac{\Gamma_1 + \Gamma_2 T_1^2 (1 + \Gamma_1^2) + \Gamma_1\Gamma_2 T_1^2 - \Gamma_1\Gamma_2 T_1^2 T_2}{1 + 2\Gamma_1\Gamma_2 T_1^2 - \Gamma_2^2 T_2^2 - 2\Gamma_1\Gamma_2 T_1 T_2^2 + \Gamma_1^2 T_2^2 T_1 - \Gamma_1^2 T_1^2 T_2}
\end{align*}
\]
Fig. 5. Real and imaginary part of complex permittivity of tap water measured in the X-band at room temperature: (1)—simultaneous method, (2)—[16], and (3)—[17].

Fig. 6. Real and imaginary part of complex permittivity of the Perspex housing determined (1)—simultaneous with tap water, (2)—simultaneous with 0.17-M saline solution, and (3)—[18]

(6a)–(6d)] is compared with that by different workers [16], [17] in Fig. 5 and show good agreement.

Because of the non-standard nature of commercial Perspex samples, we compare the permittivity of Perspex obtained simultaneously with the water, saline, and also that measured independently using the method [18] and is presented in Fig. 6.

VI. CONCLUSION

We have presented a technique of simultaneously determining the complex permittivity of two materials: a solid and a liquid, from a single measurement of S-parameters of the entire assembly. This method is useful when there is no means of prior independent measurement of the properties of the material used for the liquid housing. We have presented measured results compared with those existing in the literature to show good agreement.

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REFERENCES


